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Explicit CP Violation in the Higgs Sector of the Next-to-Minimal Supersymmetric Standard Model

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Abstract

We analyze explicit CP violation in the Higgs sector of the next-to-minimal supersymmetric standard model which contains an additional gauge singlet field N. It is shown that there is no mixing among scalar and pseudoscalar Higgs fields in the two Higgs doublets, and scalar-pseudoscalar mixings could exist between two Higgs doublets and the singlet N, and between N itself. CP symmetry is conserved in the extreme limits of $\langle N \rangle \gg v$, $\langle N \rangle \ll v$, and $\tan\beta \gg 1$. In the region of $\langle N \rangle = O(v)$ and $\tan\beta = O(1)$, large scalar-pseudoscalar mixings are realized, and this effect can reduce the lightest Higgs mass. The mass difference between no mixing and mixing case is about $10 \sim 30$ GeV. The neutron electric dipole moment in this model is consistent with the present experimental upper limit provided that squark and gaugino masses are heavy enough of $O(1)$ TeV.

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1 Introduction

The origin of CP violation is one of the most exciting topics in the present particle physics. In the standard model (SM), CP phase exists in the Kobayashi-Maskawa (KM) matrix[1], and CP is automatically conserved in the Higgs sector. However, in multi-Higgs models, CP could be violated explicitly or spontaneously in the Higgs sector[2][3]. The natural model containing two Higgs doublets is the minimal supersymmetric standard model (MSSM). The Higgs potential of the MSSM could break CP symmetry explicitly or spontaneously taking account of radiative corrections. However the radiative violation leads to the fact that explicit CP violation through the Higgs sector in the MSSM is too small to have any significant phenomenological implications and spontaneous CP violation in the MSSM also requires the existence of a light Higgs "pseudoscalar" which is inconsistent with experiments[4]. So we should extend the model in order to have significant CP violation effects in the Higgs sector in SUSY models. The minimal extension is adding the gauge singlet field N to the MSSM, which is so-called next-to-minimal supersymmetric standard model (NMSSM)[5].

In this paper we concentrate on explicit CP violation scenario in the NMSSM. Spontaneous CP violation in the NMSSM is discussed in Refs.[6][7]. Explicit CP violation in the Higgs sector is possible by complex couplings at the tree level Lagrangian. As shown later, scalars and a pseudoscalar of two Higgs doublets do not mix and scalar-pseudoscalar mixings only exist among Higgs doublets and the singlet and among the singlet itself at the tree level. These results can be derived by using the stationary conditions of phases and are completely different from the results obtained in Ref.[8]. These mixings vanish in the limits of $x \gg v$, $x \ll v$, and $\tan\beta \gg 1$, where v is the electroweak scale and x is the vacuum expectation value (VEV) of

the singlet field N. In the case of $x = O(v)$ and $\tan\beta = O(1)$, large mixings among scalars and pseudoscalars are realized in the Higgs sector. Available parameters are restricted by experiments and the lightest Higgs mass becomes smaller than the mass of the no mixing case by the effect of scalar-pseudoscalar mixings. The predicted neutron electric dipole moment (NEDM) could be consistent with the present experiment provided that squark and gaugino masses are heavy enough of $O(1)$ TeV.

Section 2 is devoted to explicit CP violation of the Higgs sector in the NMSSM. In section 3, we discuss the numerical result by using recent experimental constraints. Section 4 gives summary and discussion.

2 Explicit CP Violation in the NMSSM Higgs Sector

The superpotential of the NMSSM which contains only top Yukawa coupling for the Yukawa sector is given by

$$W = h_t Q H_2 T^c + \lambda N H_1 H_2 - \frac{k}{3} N^3. \tag{1}$$

H_1 and H_2 are Higgs doublet fields denoted as

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \tag{2}$$

with

$$H_1 H_2 = H_1^0 H_2^0 - H_1^- H_2^+. \tag{3}$$

The third generation quark doublet superfield is denoted as Q, and T^c is the right-handed top quark superfield. The coupling h_t is the top Yukawa coupling constant. One pseudoscalar which is the mixing state of H_1 and H_2 is the Goldstone boson absorbed by Z boson. There are three neutral scalars, two neutral pseudoscalars, and

one charged Higgs particle as the physical particles in the limit of no CP violation in the Higgs sector of the NMSSM.

The Higgs potential of the NMSSM derived from Eq.(1) is given by

$$V = V_{\rm nophase} + V_{\rm phase} + V_{\rm top} \ , \tag{4}$$

$$\begin{aligned}
V_{\rm nophase} &= m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + m_N^2|N|^2 \\
&\quad + \frac{g_1^2+g_2^2}{8}(|H_1|^2-|H_2|^2)^2 + \frac{g_2^2}{2}(|H_1|^2|H_2|^2 - |H_1H_2|^2) \qquad (5)\\
&\quad + |\lambda|^2[|H_1H_2|^2 + |N|^2(|H_1|^2+|H_2|^2)] + |k|^2|N|^4, \\
V_{\rm phase} &= -(\lambda k^* H_1H_2N^{*2} + \lambda A_\lambda H_1H_2N + \frac{kA_k}{3}N^3 + {\rm h.c.}) \ , \qquad (6)\\
V_{\rm top} &= \frac{3}{16\pi^2}\left[(h_t^2|H_2|^2+m_{\tilde t}^2)^2 {\rm ln}\frac{(h_t^2|H_2|^2+m_{\tilde t}^2)}{Q^2} - h_t^4|H_2|^4{\rm ln}\frac{h_t^2|H_2|^2}{Q^2}\right] \ . \qquad (7)
\end{aligned}$$

$V_{\rm top}$ represents top and stop one loop radiative corrections[9] where we assume that the soft breaking masses satisfy $m_{\tilde t_L} = m_{\tilde t_R} \equiv m_{\tilde t} \gg m_t$. The parameters λ, k, A_λ, and A_k are all complex in general. CP phase only appears in $V_{\rm phase}$ and we can remove two complex phases by the field redefinition of H_1H_2 and N[8]. Therefore, without loss of generality, we can take

$$\lambda A_\lambda > 0, \qquad kA_k > 0. \tag{8}$$

Only one phase remains in λk^* denoted as

$$\lambda k^* \equiv \lambda k e^{i\phi}, \tag{9}$$

where λ and k on the right hand side are real and positive parameters. VEVs of H_1, H_2, and N are complex in general, which should be determined by stationary conditions. We denote VEVs as

$$\langle H_1\rangle = v_1 e^{i\varphi_1}, \quad \langle H_2\rangle = v_2 e^{i\varphi_2}, \quad \langle N\rangle = x e^{i\varphi_3}, \tag{10}$$

where v_1 and v_2 are real and positive which satisfy $v \equiv \sqrt{v_1^2 + v_2^2} = 174$ GeV. Higgs fields are defined as

$$
\begin{aligned}
H_1^0 &= v_1 e^{i\varphi_1} + \frac{1}{\sqrt{2}} e^{i\varphi_1}(S_1 + i \sin\beta A), \\
H_2^0 &= v_2 e^{i\varphi_2} + \frac{1}{\sqrt{2}} e^{i\varphi_2}(S_2 + i \cos\beta A), \\
N &= x e^{i\varphi_3} + \frac{1}{\sqrt{2}} e^{i\varphi_3}(X + iY),
\end{aligned} \tag{11}
$$

where X and Y are the scalar and the pseudoscalar field of the singlet N, respectively, and $\tan\beta \equiv v_2/v_1$.

Now we calculate mass spectra. There are two physical phases of VEVs, for which we take

$$
\theta \equiv \varphi_1 + \varphi_2 + \varphi_3, \qquad \delta \equiv 3\varphi_3. \tag{12}
$$

Stationary conditions of phases

$$
\left.\frac{\partial V}{\partial \theta}\right| = 0, \qquad \left.\frac{\partial V}{\partial \delta}\right| = 0, \tag{13}
$$

induce the equations

$$
\lambda k x^2 \sin(\phi + \theta - \delta) + \lambda A_\lambda x \sin\theta = 0, \tag{14}
$$

$$
-3\lambda k v_1 v_2 \sin(\phi + \theta - \delta) + k A_k x \sin\delta = 0, \tag{15}
$$

respectively. It is noteworthy that the phase ϕ, which induces explicit CP violation in the Higgs sector, should be 0 or π from Eqs.(14) and (15) if VEVs have no phases ($\theta = \delta = 0$). Therefore we can not take $\theta = \delta = 0$ in the case of $\phi \neq 0, \pi$ contrary to the results in Ref.[8]. Three parameters $m_{H_1}^2, m_{H_2}^2$, and m_N^2 are eliminated by three stationary conditions

$$
\left.\frac{\partial V}{\partial v_i}\right| = 0 \ (i = 1, 2), \qquad \left.\frac{\partial V}{\partial x}\right| = 0. \tag{16}
$$

Then we get 5×5 neutral Higgs mass matrix

$$M^2_{H^0} = \begin{pmatrix} M^{S_1,S_2,X}_{S_1,S_2,X} & M^{A,Y}_{S_1,S_2,X} \\ & \\ (M^{A,X}_{S_1,S_2,X})^T & M^{A,Y}_{A,Y} \end{pmatrix} , \tag{17}$$

where $M^{S_1,S_2,X}_{S_1,S_2,X}$, $M^{A,Y}_{S_1,S_2,X}$, and $M^{A,Y}_{A,Y}$ are 3×3, 3×2, and 2×2 submatrices, respectively. The matrix $M^{S_1,S_2,X}_{S_1,S_2,X}$ of the scalar part of S_1, S_2, and X is given by

$$M^{S_1,S_2,X}_{S_1,S_2,X} = \begin{pmatrix} \begin{array}{l}\overline{g}^2 v^2 \cos^2\beta \\ +\lambda x A_{\sigma_1} \tan\beta\end{array} & \begin{array}{l}(\lambda^2 - \overline{g}^2/2) v^2 \sin 2\beta \\ -\lambda x A_{\sigma_1}\end{array} & \begin{array}{l}2\lambda^2 v x \cos\beta \\ -\lambda v \sin\beta A_{\sigma_2}\end{array} \\ & & \\ \begin{array}{l}(\lambda^2 - \overline{g}^2/2) v^2 \sin 2\beta \\ -\lambda x A_{\sigma_1}\end{array} & \begin{array}{l}(\overline{g}^2 + \Delta) v^2 \sin^2\beta \\ +\lambda x A_{\sigma_1}/\tan\beta\end{array} & \begin{array}{l}2\lambda^2 v x \sin\beta \\ -\lambda v \cos\beta A_{\sigma_2}\end{array} \\ & & \\ \begin{array}{l}2\lambda^2 v x \cos\beta \\ -\lambda v \sin\beta A_{\sigma_2}\end{array} & \begin{array}{l}2\lambda^2 v x \cos\beta \\ -\lambda v \cos\beta A_{\sigma_2}\end{array} & \begin{array}{l}\frac{\lambda v^2}{2x} A_\lambda \cos\theta \sin 2\beta \\ -A_k k x \cos\delta + 4k^2 x^2\end{array} \end{pmatrix} , \tag{18}$$

where $A_{\sigma_1} \equiv A_\lambda \cos\theta + kx\cos(\phi + \theta - \delta)$ and $A_{\sigma_2} \equiv A_\lambda \cos\theta + 2kx\cos(\phi + \theta - \delta)$. The Δ is derived from Eq.(7) and given as

$$\Delta \equiv \frac{3h_t^4}{4\pi^2} \ln \frac{m_t^2 + m_{\tilde{t}}^2}{m_t^2} . \tag{19}$$

The matrix $M^{A,Y}_{A,Y}$ of the pseudoscalar part of A and Y is given by

$$M^{A,Y}_{A,Y} = \begin{pmatrix} 2\lambda x A_{\sigma_1}/\sin 2\beta & \lambda v A'_\sigma \\ & \\ \lambda v A'_\sigma & \begin{array}{c}\frac{\lambda v^2}{2x} A_\lambda \sin 2\beta \cos\theta + 3A_k k x \cos\delta \\ +2\lambda k v^2 \sin 2\beta \cos(\phi + \theta - \delta)\end{array} \end{pmatrix} , \tag{20}$$

where we define $A'_\sigma \equiv A_\lambda \cos\theta - 2kx\cos(\phi + \theta - \delta)$. The scalar-pseudoscalar mixing matrix $M^{A,Y}_{S_1,S_2,X}$ is given by

$$M^{A,Y}_{S_1,S_2,X} = \begin{pmatrix} 0 & -3\lambda k v x \sin\beta \sin(\phi + \theta - \delta) \\ 0 & -3\lambda k v x \cos\beta \sin(\phi + \theta - \delta) \\ \lambda k v x \sin(\phi + \theta - \delta) & -2\lambda k v^2 \sin 2\beta \sin(\phi + \theta - \delta) \end{pmatrix} . \tag{21}$$

The mixing with scalar components S_1, S_2 and the pseudoscalar component A always vanish even if CP phase ϕ takes non-zero values. It is noted that scalar-pseudoscalar

mixings among two Higgs doublets at the tree level does not exist contrary to the Ref.[8]. In the case of $\sin(\phi+\theta-\delta)=0$, all mixing with scalars and pseudoscalars vanish.

The physical charged Higgs field is defined as $C^+ \equiv \cos\beta H^+ + \sin\beta H^{-*}$ and its mass is given by

$$m_C^2 = m_W^2 - \lambda v^2 + \frac{2\lambda A_{\sigma_1} x}{\sin 2\beta}. \tag{22}$$

Now we show parameters x and $\tan\beta$ dependence of the magnitude of scalar and pseudoscalar mixing. As for x, we consider three special limiting cases[8][10]; **(1)** $x \gg v_1, v_2$ with λ and k fixed, **(2)** $x \gg v_1, v_2$ with λx and kx fixed, and **(3)** $x \ll v_1, v_2$. And also we consider the case **(4)** with the limit of $\tan\beta \gg 1$.

(1) Limits of $x \gg v_1, v_2$ (λ and k fixed);

In this limit with $A_\lambda, A_k = O(x)$, $\sin\delta$ goes to zero from Eq.(15). M_X^X and M_Y^Y elements are of $O(x^2)$ because of $\cos\delta \simeq 1$ and mixing components with S_1, S_2, A and X, Y are of $O(vx)$. Then the mixing angles of scalars and pseudoscalars become $O(v/x)$. Therefore X and Y are heavy enough to decouple from S_1, S_2, and A, and it is enough to consider the 3×3 submatrix of $S_1 - S_2 - A$ to estimate CP violation effects. We know that S_1, S_2, and A do not mix from Eq.(21). Then CP violation in the Higgs sector vanishes in this limit.

(2) Limits of $x \gg v_1, v_2$ (λx and kx fixed);

This limit with $A_\lambda, A_k = O(\lambda v)$ reduces the NMSSM to the MSSM. We can easily show that all scalar-pseudoscalar mixing elements in Eq.(21) vanish. Then CP symmetry in the Higgs sector restores in this limit.

(3) Limits of $x \ll v_1, v_2$;

In this limit with $A_\lambda, A_k = O(v)$, $\sin(\phi+\theta-\delta)$ goes to zero from Eq.(15). Then

all components of $M^{A,Y}_{S_1,S_2,X}$ vanish and the Higgs potential has CP symmetry in this limit.

(4) Limits of $\tan\beta \gg 1$;

In the large $\tan\beta$ limit, S_2-Y and X-Y components in the $M^{A,Y}_{S_1,S_2,X}$ vanish. Then it is enough to consider the S_1-Y and S_2-X-A submatrices. These are

$$M^{S_1,Y}_{S_1,Y} = \begin{pmatrix} \lambda x A_{\sigma_1}\tan\beta & -3\lambda kvx\sin(\phi+\theta-\delta) \\ -3\lambda kvx\sin(\phi+\theta-\delta) & 3A_k kx\cos\delta \end{pmatrix}, \tag{23}$$

and

$$M^{S_2,X,A}_{S_2,X,A} = \begin{pmatrix} (\overline{g^2}+\Delta)v^2 & 2\lambda^2 vx & 0 \\ 2\lambda^2 vx & -A_k kx\cos\delta + 4k^2x^2 & 2\lambda kvx\sin(\phi+\theta-\delta) \\ 0 & 2\lambda kvx\sin(\phi+\theta-\delta) & 2\lambda x A_{\sigma_1}/\sin 2\beta \end{pmatrix}, \tag{24}$$

respectively. Only S_1-S_1 and A-A components are dominant in each matrix unless $A_{\sigma_1}=0$. Then scalar-pseudoscalar mixings become negligibly small and CP violation in the Higgs sector vanishes in this limit. If $A_{\sigma_1}=0$, the phase ϕ should be 0 or π from Eq.(14) since δ goes to be 0 or π from Eq.(15). It is not the case of explicit CP violation of the Higgs sector in the NMSSM.

In the above various limits of **(1)**$\sim$**(4)**, CP symmetry is approximately conserved in the Higgs sector. As for the region of $x=O(v)$ and $\tan\beta=O(1)$, large scalar-pseudoscalar mixings are realized. We will discuss this case numerically in the next section.

3 Numerical Analysis of Explicit CP Violation

In this section we show numerical examples to realize the large CP violation. We discuss the case of $x=O(v)$ and $\tan\beta=O(1)$.

In Fig.1 we show the experimentally allowed region in the $\cos\phi-\lambda$ plane for fixed values of other parameters. The values of parameters are

$$
\begin{aligned}
& k = 0.1, \qquad \tan\beta = 2, \qquad m_{\tilde{t}} = 3 \text{ TeV}, \\
& A_k = v, \qquad A_\lambda = v/2, \qquad x = 5\ v.
\end{aligned} \tag{25}
$$

Here we consider the following experimental constraints[7];

(A) The lightest Higgs boson (h_1) and the second lightest Higgs boson (h_2) have not been observed in the decay of Z. So the condition of

$$
m_{h_1} + m_{h_2} > m_Z
$$

should be satisfied kinematically, or in the case of the sum of m_{h_1} and m_{h_2} is smaller than m_Z, the branching ratio $B(Z \to h_1 h_2)$ should be less than 10^{-7}[11]. The decay rate is

$$
\Gamma(Z \to h_1 h_2) = \frac{M_Z}{16\pi} g^2_{Zh_1h_2} \lambda^{\frac{3}{2}}(1, x_1, x_2), \tag{26}
$$

where $\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and $x_i \equiv m^2_{h_i}/M^2_Z$. The coupling of $g_{Zh_1h_2}$ is defined as

$$
g_{Zh_1h_2} \equiv g_2[\cos\beta(a_{1S_2}a_{2A} - a_{2S_2}a_{1A}) - \sin\beta(a_{1S_1}a_{2A} - a_{2S_1}a_{1A})], \tag{27}
$$

where a_{iJ} is the ratio of the field J component in h_i ($i = 1, 2$).

(B) The Higgs h_i has not been observed in the decay $Z \to h_i + Z^* \to h_i + l^+l^-$ [12]. So $B(Z \to h_i l^+ l^-)$ should be smaller than 1.3×10^{-7}[11]. In the case m_{h_1} and/or m_{h_2} being smaller than M_Z, the decay rate becomes

$$
\begin{aligned}
\Gamma(Z \to h_i l^+ l^-) \quad = \quad & \frac{1}{96\pi^3} \frac{g^2_{ZZh_i} g^2_{Zl^+l^-}}{M_Z} (|C_L|^2 + |C_R|^2) \\
& \int_{\rho_i}^{\frac{1+\rho_i^2}{2}} \frac{1 + \rho_i^2 - 2x}{(\rho_i^2 - 2x)^2 + \Gamma_Z^2/(4M_Z^2)} (x^2 - \rho_i^2)^{1/2} dx,
\end{aligned} \tag{28}
$$

where $\rho_i = m_{h_i}/M_Z$, $x = E_{h_i}/M_Z$, $g_{Zl^+l^-} = 2e/\sin 2\theta_W$, $C_L = -\frac{1}{2} + \sin^2\theta_W$ and $C_R = \sin^2\theta_W$. The coupling constant g_{ZZh_i} is given by

$$g_{ZZh_i} \equiv \frac{g_2}{2\cos\theta_W} M_Z \cos\beta(a_{iS_1} + a_{iS_2}\tan\beta). \tag{29}$$

(C) The chargino $\tilde{\chi}^{\pm}$ has not been observed in the decay $Z \to \tilde{\chi}^+\tilde{\chi}^-$, the mass of chargino $M_{\tilde{\chi}^{\pm}}$must be larger than 45.2 GeV[11].

The μ term in the MSSM is represented as λx in the NMSSM, so the chargino mass becomes small as λ becomes small. With the wino soft breaking mass M_2 being 1 TeV, the dashed boundary in the figure is derived from the constraint **(C)**.

In Fig.2 the experimentally allowed region of $\lambda-\tan\beta$ plane is shown at $\cos\phi = 0$, where we take other parameters as the same in Eq.(25). The allowed region of λ is about $0.06 \sim 0.24$ which has small dependence on $\tan\beta$. We only consider the region of $\tan\beta \geq 1$ in this paper[13].

In Fig.3 the mass of the lightest Higgs particle versus λ is shown. The region of λ is limited from constraints **(A)**$\sim$**(C)**. CP violation in the Higgs sector requires scalar-pseudoscalar mixing, which reduces the magnitude of the mass up to $10 \sim 30$ GeV. The qualitative result is not changed in the region of $m_{\tilde{t}} = 1 \sim 3$ TeV.

Now we discuss the NEDM. The phases which contribute to the NEDM at one loop level are induced from the chargino, the neutralino, and the squark mass matrices. As following the analysis by Kizukuri and Oshimo[14], the chargino diagram gives the dominant contribution to the NEDM under the GUT relation of gaugino soft masses. The NEDM is proportional to λ and $\sin(\theta + \eta)$, where η is the phase of M_2. Then the maximal NEDM occurs at $\lambda = 0.27$ and $\sin(\theta + \eta) = 1$. The calculated value of the NEDM is 3.0×10^{-27} e$\cdot$cm with $M_2 = 1$ TeV and squark masses $m_{\tilde{u},\tilde{d}} = 3$ TeV. This value is about two orders smaller than the experimental

upper bound[11]. Squark masses should be of $O(1)$ TeV to retain consistency with the EDM experiment. For example, in the case of $\lambda = 0.23$ which satisfy constraints **(A)**$\sim$**(C)**, squark masses must be larger than 1.4 TeV.

4 Summary and Discussion

We have studied explicit CP violation in the Higgs sector of the NMSSM. Mixings with scalars and a pseudoscalar of two Higgs doublets do not exist by the stationary conditions of phases. Scalar-pseudoscalar mixings only exist through the singlet field N. This situation is not changed drastically by radiative corrections because loop effects, which contribute scalar-pseudoscalar mixings, are negligibly small as same as the case of explicit CP violation in the MSSM. These mixings would vanish in the limits of $x \gg v$, $x \ll v$, and $\tan\beta \gg 1$. In the region of $x = O(v)$ and $\tan\beta = O(1)$, CP violating effects are realized in the Higgs sector. The lightest Higgs mass is reduced about $10 \sim 30$ GeV by the effect of scalar-pseudoscalar mixings. The value of the NEDM predicted by this model is consistent with the experiment provided that squark and gaugino masses are heavy enough of $O(1)$ TeV.

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References

[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] T. D. Lee, Phys. Rev. **D8**, 1226 (1973).

[3] S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976);
G. C. Branco, Phys. Rev. Lett. **44**, 504 (1980).

[4] N. Maekawa, Phys. Lett. **B282**, 387 (1992);
A. Pomarol, Phys. Lett. **B287**, 331 (1992).

[5] P. Fayet, Nucl. Phys. **B90**, 104 (1975);
R. K. Kaul and P. Majumdar, Nucl. Phys. **B199**, 36 (1982);
R. Barbieri, S. Ferrara and C. A. Sávoy, Phys. Lett. **119B**, 343 (1982);
H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. **120B**, 346 (1983);
J. M. Frère, D. R. T. Jones and S. Raby, Nucl. Phys. **B222**, 11 (1983);
J. P. Derendinger and C. A. Savoy, Nucl. Phys. **B237**, 307 (1984).

[6] K. S. Babu and S. M. Barr, Phys. Rev. **D49**, 2156 (1994).

[7] N. Haba, M. Matsuda, and M. Tanimoto, hep-ph/9512421.

[8] M. Matsuda and M. Tanimoto, Phys. Rev. **D52**, 3100 (1995).

[9] S. Coleman and E. Weinberg, Phys. Rev. **D7**, 1888 (1973);
S. Weinberg, Phys. Rev. **D7**, 2887 (1973).

[10] J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner, Phys. Rev. **D39**, 844 (1989).

[11] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. **D50**, 1173 (1994).

[12] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. **B313**, 312 (1993).

[13] See,e.g., G. F. Giudice and G. Ridolfi, Z. Phys. **C41**, 447 (1988);
M. Olechowski and S. Pokorski, Phys. Lett. **B214**, 393 (1988);
M. Drees and M. M. Nojiri, Nucl. Phys. **B369**, 54 (1992).

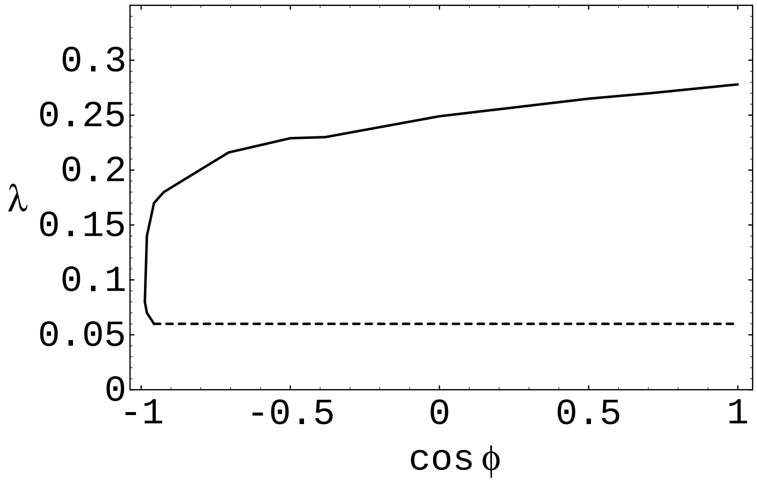
[14] Y. Kizukuri and N. Oshimo, Phys. Rev. **D46**, 3025 (1992).

Figure Captions

Fig.1 The allowed region in the $\cos\phi - \lambda$ plane for $k = 0.1, \tan\beta = 2, m_{\tilde{t}} = 3$ TeV, $A_k = v$, $A_\lambda = v/2$, and $x = 5v$. The solid boundary corresponds to constraints **(A)** and **(B)**. The dashed boundary corresponds to the constraint **(C)**.

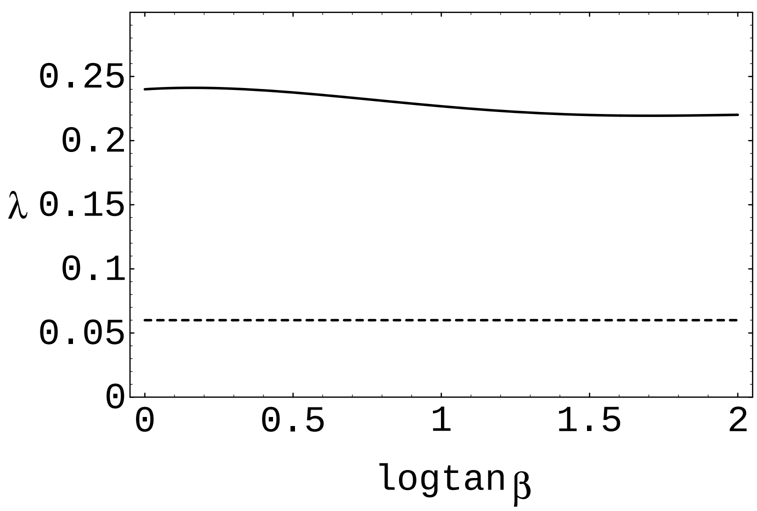
Fig.2 The allowed region in the $\tan\beta - \lambda$ plane at $\cos\phi = 0$. Other parameters are given in Eq.(25) and the boundaries are obtained by the same constraints in Fig.1.

Fig.3 λ dependence of the predicted lightest Higgs particle mass; short-dashed line: $\cos\phi = 1$ (no mixing case), solid line: $\cos\phi = 1/2$, long-dashed line: $\cos\phi = 0$, long-dashed-dotted line: $\cos\phi = -1/2$.



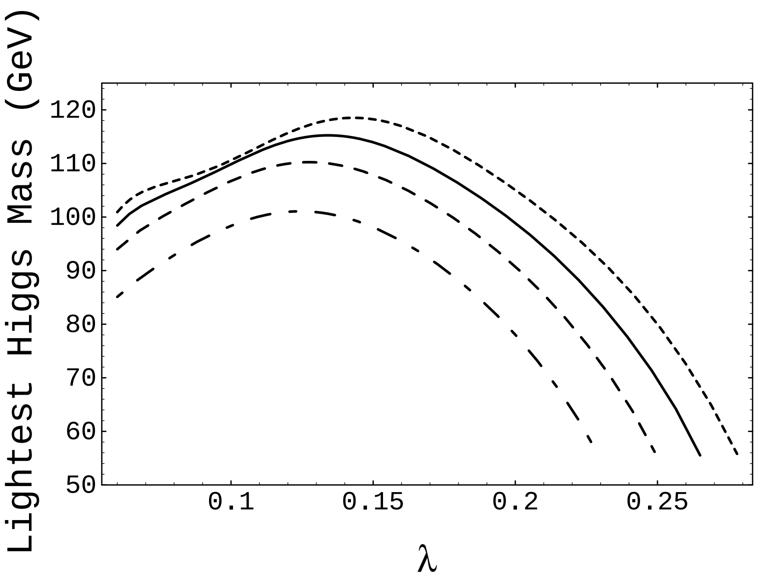
0.3
0.25
0.2
λ
0.15
0.1
0.05
0
-1
-0.5
0
0.5
1
cosϕ

Fig.1



0.25
0.2
λ
0.15
0.1
0.05
0
0
0.5
1
1.5
2
logtanβ

Fig.2



Lightest Higgs Mass (GeV)
120
110
100
90
80
70
60
50
0.1
0.15
0.2
0.25
λ

Fig.3